VI Congreso Galego de Estatística e Investigación de Operacións
Vigo 5–7 de Novembro de 2003

Procedurally Fair and Stable Matching

Bettina Klaus[1], Flip Klijn[2]

[1]Departament d'Economia i d'Història Econòmica
Universitat Autònoma de Barcelona
[2]CODE and Departament d'Economia i d'Història Econòmica
Universitat Autònoma de Barcelona

RESUMO

We study procedurally fair matching mechanisms that produce stable matchings for the so-called marriage model of one-to-one, two-sided matching. Our main focus is on two such mechanisms: *employment by lotto* introduced by Aldershof *et al.* (1999) and the *random order mechanism* due to Roth and Vande Vate (1990) and Ma (1996). For both mechanisms we give various examples of probability distributions on the set of stable matchings and discuss properties that differentiate employment by lotto and the random order mechanism. Furthermore, we correct some misconceptions by Aldershof *et al.* (1999) and Ma (1996) that exist on the probability distribution induced by both mechanisms. Finally, we consider an adjustment of the random order mechanism, *the equitable random order mechanism.*

Palabras e frases chave: procedural fairness, random mechanism, stability, two-sided matching.

1. INTRODUCCIÓN

The marriage model describes a two-sided matching market without money where the two sides of the market for instance are workers and firms (job matching) or medical students and hospitals (matching of students to internships). We use the common terminology in the literature and refer to one side of the market as "men" and to the other as "women." An outcome for a marriage market is called a matching, which can simply be described by a collection of single agents and "married" pairs (consisting of one man and one woman). Loosely speaking, a matching is stable if all agents have acceptable spouses and there is no couple whose members both like each other better than their current spouses. Gale and Shapley (1962) formalized this notion of stability for marriage markets and provided an algorithm to calculate stable matchings. These classical results (Gale and Shapley, 1962) inspired many researchers to study stability not only for the marriage model, but for more general models as well. We refer to Roth and Sotomayor (1990) for a comprehensive account on stability for two-sided matching models.

In this paper we study a combination of fairness and stability in the marriage model. Masarani and Gokturk (1989) showed several impossibilities to obtain a fair deterministic matching mechanism within the context of Rawlsian justice. Therefore, we opt for an approach of procedural fairness. Since for any deterministic matching mechanism we can detect an inherit favoritism either for one side of the market or for some agents over others, in order to at least recover *ex ante* fairness, we consider probabilistic stable matching mechanisms that assign to each marriage market a probability distribution over the set of stable matchings. We do not intend to judge the fairness of a probabilistic stable matching mechanism by judging the assigned probability distributions, but by considering procedurally fair matching algorithms in which the sequence of moves for the agents is drawn from a uniform distribution. Hence, whenever an agent has the same probability to move at a certain point in the procedure that determines the final probability distribution, we consider the random stable matching mechanism to be *procedurally fair*. In other words, here we focus on "procedural justice" rather than on "endstate justice" (Moulin, 1997 and Moulin, 2003).

First, we analyze a random matching mechanism proposed by Aldershof *et al.* (1999) called *employment by lotto*. Loosely speaking employment by lotto can be considered to be a random serial dictatorship on the set of stable matchings. A first agent is drawn randomly and can discard all stable matchings in which he/she is not matched with his/her best partner in a stable matching. Exclude the first agent and his/her partner from the set of agents and randomly choose the next agent who can discard all stable matchings in which he/she is not matched with his/her best partner in the reduced set of stable matchings. Continue with this sequential reduction of the set of stable matchings until it is reduced to a singleton. Using all possible sequences of agents, this mechanism induces a probability distribution on the set of stable matchings. The associated probabilistic matching mechanism of this probabilistic sequential dictatorship equals employment by lotto. We give various examples of probability distributions on the set of stable matchings induced by employment by lotto, show certain limitations of this mechanism (*e.g.*, complete information of all agents' preferences is needed), and disprove several conjectures about the distribution of probabilities made in Aldershof *et al.* (1999).

Next, we consider a random matching mechanism based on Roth and Vande Vate's (1990) results. We follow Ma (1996) and refer to this rule as the *random order mechanism*. The basic idea is as follows. Imagine an empty room with one entrance. At the beginning, all agents are waiting outside. At each step of the algorithm, one agent is chosen randomly and invited to enter. Before an agent enters the matching in the room is stable. However, once an agent enters the room, the existing matching in the room may become unstable, meaning that the new agent can form a blocking pair with another agent that already is present in the room. By satisfying this (and possible subsequent) blocking pair(s) in a certain way a new stable matching including the entering agent is obtained for the marriage market in the room. After a finite number of steps a stable matching for the original marriage market is obtained. Using all possible sequences of agents, this mechanism induces a probability distribution on the set of stable matching. The associated probabilistic matching mechanism equals the random order mechanism. We give various examples of probability distributions on the set of stable matchings induced by the random order mechanism. Furthermore, we show that the probability distribution Ma (1996) presents is not correct. The mistake in the calculations by Ma (1996) is due to the fact that even though the example looks very symmetric, some of the calculations are not as "symmetric" due to the fact that the random order mechanism does not satisfy what we call *independence of dummy agents*; that is, the final probability distribution on the set of stable matchings may crucially depend on preferences of agents who are matched to the same partner in all stable matchings. Moreover, we answer in the negative a question posed by Cechlárová (2002) on whether certain matchings can always be reached.

Finally, following a suggestion by Romero-Medina (1995), we briefly discuss an adjustment of the random order mechanism, *the equitable random order mechanism*. This mechanism limits the set of options available for each agent, trying to avoid the inherent favoritism of optimal matchings. We show that even for small markets the three mechanisms may give completely different and somewhat surprising outcomes.

In all our examples, we implement the mechanisms discussed so far in Matlab ©. In some examples the resulting probabilities are rounded.

2. AGRADECEMENTOS

The work of the authors is partially supported by Research Grant BEC2002-02130 from the Spanish Ministry of Science and Technology. B. Klaus's research is supported by a Ramón y Cajal contract of the Spanish Ministry of Science and Technology. F. Klijn's research is supported by a Marie Curie Fellowship of the European Community programme "Improving Human Research Potential and the Socio-economic Knowledge Base" under contract number HPMF-CT-2001-01232.

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