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COLUMN GENERATION BASED HEURISTIC FOR A VEHICLE ROUTING PROBLEM WITH 2-DIMENSIONAL LOADING CONSTRAINTS: A PROTOTYPE

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ABSTRACT

In this paper, we propose a new approach for a vehicle routing problem with 2-dimensional loading constraints. Our approach relies on a column generation based heuristic which can be described in two distinct phases. In the first phase, strategies are proposed to generate initial columns for the restricted master problem corresponding to valid routes. In the second phase, the restricted master problem provides information for the solution of a relaxed subproblem. The aim of this subproblem is to find attractive columns, which are in fact routes. A bottom-left heuristic is also proposed to validate the obtained routes both in terms of the packing and the sequencing constraints. A route shortening process is described to build a feasible route for the cases in which the bottom-left heuristic is not successful.

Keywords: Vehicle routing problem, packing constraints, sequencing constraints, column generation, heuristics.

1. VEHICLE ROUTING PROBLEM WITH 2-DIMENSIONAL LOADING CONSTRAINTS

The Vehicle Routing Problem with Loading constraints (L-CVRP) is a recent and challenging optimization problem which integrates two hard combinatorial problems: the capacitated vehicle routing problem and the bin packing problem either in two- or three-dimensional case (2L-CVRP or 3L-CVRP, respectively).

The L-CVRP has motivated a growing interest in the scientific community in the recent past, inspiring works which include additional features. One of these features is the relation between the loading operation of the vehicles and the sequence through which the customers will be visited. In practice, some approaches consider that unloading items in one customer does not require to move items of customers who have not already served. This problem is known as a sequential L-CVRP. Other features may include the orientation of the items, which can be fixed or variable [Fuellerer et al., 2009], the association of time windows [Moura, 2008, Moura and Oliveira, 2009] or pickup and delivery [Malapert et al., 2008].

In this paper, we focus on the capacitated vehicle routing problem with 2-dimensional loading constraints (2L-CVRP). Due to its complexity, the vast majority of approaches for the L-CVRP consists in heuristics like tabu search [Gendreau et al., 2008], guided tabu search [Zachariadis et al., 2009], extended guided tabu search [Leung et al., 2011] or ant colony optimization [Fuellerer et al., 2009]. In 2011, good results were achieved by a sophisticated multi-start evolutionary local search [Duhamel et al., 2011]. To the best of our knowledge, only one exact approach was proposed in the literature in 2007 [Iori et al., 2007]. This approach relies on a branch-and-cut method. Some separation procedures are used in order to find violated inequalities. Additionally, the authors proposed a branch-and-bound algorithm to check whether the packing is valid. The proposed model will be discussed in Section 3.

The complexity of 2L-CVRP motivates the presented approach which relies on the Dantzig-Wolfe decomposition [Dantzig and Wolfe, 1960]. The method is based on a reformulation of the original problem and the decomposition of the original model into one restricted master problem and one or more subproblems. The restricted master problem has less constraints when compared to the original model. The Dantzig-Wolfe decomposition may provide stronger models than the original formulations from which these models are obtained. However, the number of variables can be exponential. To overcome this, column generation can be used by choosing a restricted set of columns for the restricted master problem. The optimal solution of this initial problem will provide information to the subproblem or subproblems in order to find the column with the most negative reduced cost, if it exists. If a negative value is obtained, the column suggested by the subproblem is added to the restricted master problem. Then, the restricted master problem is reoptimized. The process is repeated until no more negative reduced cost columns can be found.

This paper is organized as follows. In Section 2 we present the description of the problem. In Section 3, we describe a two-index flow model for the 2L-CVRP. In Section 4, the approach which motivates this paper is presented. Section 5 draws some final conclusions.

2. PROBLEM DESCRIPTION

In this paper we address a 2L-CVRP which is defined as follows. A set V represents the set composed of n nodes which includes the depot (vertex 0) and $n-1$ customers. The fleet is homogeneous, and it is composed by K vehicles, with a capacity C, and an available rectangular loading surface with height H and width W.

Each customer has 2-dimensional items, which must be placed in the vehicles with fixed orientation. The overall items packed into the vehicle must not exceed its capacity and the vehicle must be loaded in such a way that unloading the items for a given customer does not require to move other items in the vehicle for the following customers. Obviously, and as it happens in all bin packing problems, the items must not overlap, and must be completely included in the surface.

The cost of travelling along an edge which links customers i and j is represented by c_{ij}. The objective is to serve all the customers in a single visit, i.e. the demand can not be split, while minimizing costs.

3. INTEGER PROGRAMMING MODELS

3.1. Integer programming model for the 2L-CVRP

In this section, we present a 2L-CVRP formulation proposed in [Iori et al., 2007]. This formulation relies on the two-index flow variables, considering a set of nodes V, a set of edges E and K vehicles. The cost of travelling along the edge $e \in E$ is denoted as c_e. The decision variables z_e are binary and take the value one if a vehicle uses edge e in its route, and value zero otherwise. Additionally, $\delta(S)$ represents the set of edges with one endpoint in S and the other in $V \backslash \{S\}$, σ represents the bijection which defines the order by which the customers will be visited, and $\Sigma(S)$ represents the collection of sequences σ in which (S,σ) is a feasible route. Finally, *r(S)* represents the minimum number of vehicles required to serve all the customers in S.

$$\min \quad \sum_{e \in E} c_e z_e \tag{1}$$

$$\text{s.t.} \quad \sum_{e \in \delta(i)} z_e = 2 \quad \forall i \in V \backslash \{0\} \tag{2}$$

$$\sum_{e \in \delta(0)} z_e = 2K \tag{3}$$

$$\sum_{e \in \delta(S)} z_e \geq 2r(S) \quad \forall S \subseteq \in V \backslash \{0\}, S \neq \emptyset \tag{4}$$

$$\sum_{e \in E(S,\sigma)} z_e \leq |S| - 1 \quad \forall (S, \sigma) \quad s.t. \sigma \notin \Sigma(S) \tag{5}$$

$$z_e \in \{0, 1\} \quad \forall e \in E \tag{6}$$

The strategy used to find the integer optimal is as follows. Starting from the problem formulation (1)-(6), the set of constraints (4),(5) and (6) are relaxed. The resulting problem is solved and then heuristic separation procedures are applied in order to find violated inequalities concerning the set of constraints (4) and (5). In this last case, the separation procedure is composed by a branch-and-bound algorithm to check the feasibility of a given route.

The 2-dimensional orthogonal packing problem with unloading constraints consists in a subproblem of the problem referred to above. A branch-and-cut approach for this problem was recently proposed in [Côté et al., 2013]. Indeed, this problem considers the packing and sequencing constraints. Therefore, this approach is a relaxation of the model (1)-(6) since it excludes the route component. For each node of the branching tree, a modified one dimensional contiguous bin packing problem is solved.

3.2. Set partitioning model

It is possible to apply a Dantzig-Wolfe decomposition to the 2L-CVRP and consequently have a set-partitioning structure. For this purpose, let Ω be the set of all feasible routes. For each route $r \in \Omega$ one decision variable defined by y_r is associated which takes value one if the route is included in the solution, and zero otherwise. In this formulation each column corresponds to a feasible route which visits a subset of customers by a given sequence, and takes into account the sequencing constraints and the vehicle constraints through the restriction to K vehicles from a homogeneous fleet.

Each route r with a total cost c_r can be described by a vector $(a_{1r},\ a_{2r}, ..., a_{n-1,r})^T$ being a_{ir} the coefficient which indicates if customer i is visited in route r for $i \in \{1, ..., n-1\}$. Note that since more than one route can visit the same subset of customers, it is possible that two or more columns may be defined by the same vectors. However, the sequences of that route are not equal and consequently the cost of the two routes should be different.

$$\min \quad \sum_{r \in \Omega} c_r y_r \tag{7}$$

$$\text{s.t.} \quad \sum_{r \in \Omega} a_{ir} y_r = 1, \quad \forall i \in \{1, ..., n-1\} \tag{8}$$

$$\sum_{r \in \Omega} y_r \leq K \tag{9}$$

$$y_r \in \{0, 1\} \tag{10}$$

4. A COLUMN GENERATION BASED HEURISTIC

4.1. Outline

In this section, we describe the general elements of a column generation based heuristic for the 2L-CVRP. The restricted master problem is initialized with a limited number of columns. Some strategies may be applied to generate initial columns, as we will explain in the next subsection. The LP relaxation of the restricted master problem is then solved, thus providing information concerning the dual variables associated to each constraint. This information is used by one relaxed subproblem which has associated some heuristic procedures in order to derive an attractive column. Then, the attractive column, if any, is incorporated in the restricted master problem.

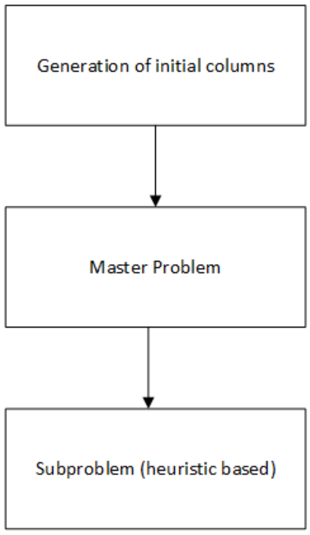

Figure 1: Overall approach.

4.2. Strategies for the generation of initial columns

The restricted master problem is always composed by valid columns which represent feasible routes defined by a set of customers and a sequence to visit them, such that the capacity and the sequencing constraints are satisfied. In order to ensure the validity of the restricted master problem, an artificial column can be used, assigning all customers to this column with a huge cost. However a heuristic can be used since it provides a valid solution to the 2L-CVRP. The columns of the restricted master problem are thus built using this valid solution.

In this paper, we propose a different approach by grouping customers according to some criteria. Different strategies can be used to group a subset of customers and define a sequence. Regardless of the strategy, at the end of the process, a route must be defined, i.e., a subset of customers and a sequence to visit them must be known. Two strategies are explored using different criteria in order to create different partitions of customers. While the first strategy starts with a given subset of customers, the second strategy iteratively constructs a feasible route. The strategies are defined below.

(i) Generation of subsets S_i of p customers from $V \backslash \{0\}$, $|S_i| = m$, taking into account the criteria:

- The total weight of the items which compose the demand of the customers from a subset S_i does not exceed a given percentage of the capacity of the vehicle;
- The total area of the items which compose the demand of the customers from a subset S_i does not exceed a given percentage of the area of the vehicle.

(ii) Generation of n sequences. Each sequence R_i of customers begin at the depot. Then, one customer j is iteratively selected taking into account the following criteria:

- Customer j is the nearest customer;
- Customer j and the customers of R_i totalize a weight demand which does not exceed a given percentage of the capacity of the vehicle;
- Customer j and the customers of R_i totalize a area demand which does not exceed a given percentage of the area of the vehicle.

Note that if strategy (i) is chosen, it is necessary to define a sequence through which the customers are visited, always beginning and ending at the depot. These criteria potentiate the satisfaction of the capacity and sequencing constraints.

After a sequence (i.e., a route) is defined, it is important to test if it is valid. Thus, lower bounds for the bin packing problem are calculated based on the customers which compose that route. If the lower bound is greater than one, the route is invalid. Otherwise, a bottom-left heuristic with sequencing constraints is applied. If with this heuristic a valid packing is derived, the column which represents that route is added to the master problem. It is important to note that since this bottom-left approach is heuristic, it is possible to exclude routes that are valid, but not found heuristically. Consequently, the present columns in the master problem may not be sufficient to represent an integer valid solution for the 2L-CVRP with a maximum of K routes, excluding the case of the artificial column referred to above.

4.3. Strategies for the generation of attractive columns

Since the partitioning constraints and vehicle constraint are in the master problem, the capacity and sequencing constraints are assigned to the subproblem. The subproblem corresponds to a Elementary Shortest Path Problem with Capacity Constraints (ESPPCC), which is NP-hard. The purpose is to find the column with the most negative reduced cost. Let π_i with $i \in V \backslash \{0\}$ be the dual variables of the restricted master problem associated to the set of constraints (8), and μ the dual variable associated to the maximum number of vehicles in the constraint (9). The expression of the reduced cost c'_r for a given route $r \in \Omega$ is given by:

$$c'_r = c_r - \sum_{i \in V \backslash \{0\}} \pi_i a_{ir} - \mu \tag{11}$$

Considering C_r the sequence of arcs (i,j) in route r, $C_r = \{(0,i_1),(i_1,i_2),...,(i_n,0)\}$, the reduced cost can be reformulated as follows:

$$c'_r = \sum_{(i,j) \in C_r, i \neq 0} (c_{ij} - \pi_i) + c_{0i_1} - \mu \tag{12}$$

Additionally, consider P the set of feasible routes taking into account the sequencing, capacity and packing constraints. Thus, and aiming to find the column with the most negative reduced cost, the subproblem is formulated as follows.

$$\begin{aligned}
\min \quad & \sum_{(i,j) \in A} c'_{ij} x_{ij} && (13)\\
\text{s.t.} \quad & \sum_{i \in V \backslash \{0\}} x_{0i} = 1 && (14)\\
& \sum_{i \in V \backslash \{0\}} x_{i0} = 1 && (15)\\
& \sum_{i \in V} x_{ij} - \sum_{i \in V} x_{ji} = 0, \qquad \forall j \in V \backslash \{0\} && (16)\\
& \mathrm{X} = \{x_{ij} | (i,j) \in A\} \in P && (17)\\
& x_{ij} \in \{0,1\} \qquad \forall (i,j) \in A && (18)
\end{aligned}$$

where

$$c'_{ij} = \begin{cases} c_{ij} - \pi_i, & \forall (i,j) \in A, i \neq 0 \\ c_{ij} - \mu, & \forall (0,j) \in A \end{cases}$$

Note that as referred to above, the subproblem is NP-hard. Consequently, in this approach the constraint (17) is relaxed and a capacity constraint is added to the subproblem (13)–(18):

$$\sum_{i \in V} w_i \sum_{(i,j) \in A} x_{ij} \leq W \tag{19}$$

The strategy consists in solving this relaxed subproblem exactly in order to find, if it exists, an attractive column. The attractive column will suggests one more partition of customers to be visited and the sequence in which the visits will occur. Then it is necessary to evaluate if this partition will compose a valid packing while satisfies the sequencing and capacity constraints. Then, lower bounds for the bin packing problem are applied in order to avoid the useless generation of an invalid packing. If the calculated lower bound is greater than one, the obtained route is invalid, and thus it is necessary to drop one or more customers from this route, as explained later. However, if the lower bound is not greater than one, a bottom-left heuristic with sequencing constraints is applied. Then two distinct scenarios are possible:

- The partition corresponds to a valid packing, and then this column is associated to the master problem;
- The bottom-left with sequencing constraints is not able to achieve a valid packing.

If a valid packing is not achieved, one or more customers are dropped from the obtained route. This means that one customer at time will be dropped from the sequence by which the customers are served. This strategy will try to derive a valid packing through the elimination of some items. In order to choose one customer to eliminate from the sequence, some criteria can be used:

- The customer with the highest number of items;
- The customer with the highest value of the sum of the areas of their items;
- The customer with the lowest value of the reduced cost π_i.

In this paper, this last strategy was selected. The route generated by the subproblem will exclude the customer whose reduced cost is less significant, in order to avoid a significant modification in the value of the reduced cost of the optimal solution given by the subproblem. The bottom-left heuristic with sequencing constraints is repeated to the remaining sequence. Then two distinct scenarios are possible:

- The bottom-left heuristic performs a valid packing to the new partition;
- The reduction of the route is insufficient to generate a valid packing, since in the worst case one customer has always a valid packing.

If a valid packing is again impossible to derive, a new customer with the lowest reduced cost is removed from the route, and the bottom-left heuristic with sequencing constraints is applied until a valid packing is achieved or while the reduced cost remain negative. This last case is due to the fact that removing one or more customers from the expression of reduced costs may induce a non-negative value of the expression of reduced cost. If this happens, the corresponding column cannot be associated to the master problem. In this case, the master problem is solved to optimality by a solver. The procedure described in this section is summarized in Figure 2.

However, it may happen that it is not possible to find an integer solution with the actual columns of the master problem. In order to solve this problem, one of two strategies can be applied:

- The value of p in the strategy (i) related in the previous section is increased in order to combine more customers and possibly derive better solutions;
- The value of m in the strategy (i) related in the previous section is increased in order to possibly generate more columns;
- The linear relaxation of the master problem is applied. The value of the decision variables is rounded in order to attempt to derive a valid and integer solution.

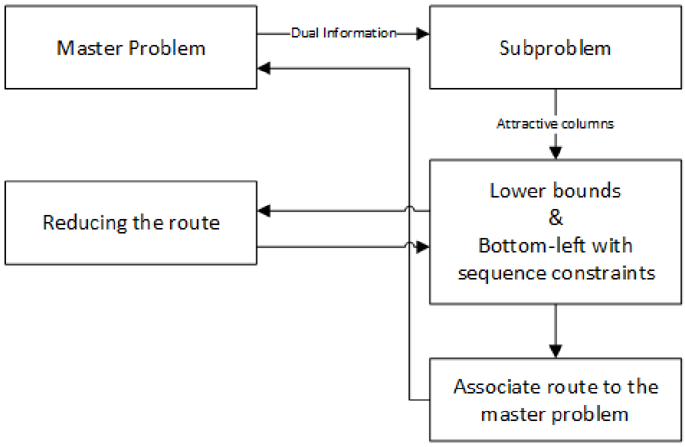

Figure 2: Generation of attractive columns.

5. CONCLUSIONS

In this paper a column generation based heuristic approach was presented, concerning the capacitated vehicle routing problem with two-dimensional loading constraints. This problem is NP-hard. Only one exact approach is related in the literature concerning this problem. This paper intends to present an alternative approach through a prototype model of a column generation based heuristic.

The contributions of this paper are mainly the strategies for generating initial columns, the proposal of a relaxed subproblem which is solved with heuristic support and finally, for those cases in which the obtained route can not generate a valid packing, a simple route shortening process is also suggested.

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REFERENCES

Côté, J.-F., Gendreau, M., and Potvin, J.-Y. (2013). An exact algorithm for the two-dimensional orthogonal packing problem with unloading constraints.

Dantzig, G. B. and Wolfe, P. (1960). Decomposition principle for linear programs. Operations Research, 8(1):101-111.

Duhamel, C., Lacomme, P., Quilliot, A., and Toussaint, H. (2011). A multi-start evolutionary local search for the two-dimensional loading capacitated vehicle routing problem. Computers & Operations Research, 38(3):617-640.

Fuellerer, G., Doerner, K. F., Hartl, R. F., and Iori, M. (2009). Ant colony optimization for the two-dimensional loading vehicle routing problem. Computers & Operations Research, 36(3):655-673.

Gendreau, M., Iori, M., Laporte, G., and Martello, S. (2008). A tabu search heuristic for the vehicle routing problem with two-dimensional loading constraints. Networks, 51(1):4-18.

Iori, M., Salazar-Gonzalez, J.-J., and Vigo, D. (2007). An exact approach for the vehicle routing problem with two-dimensional loading constraints. Transportation Science, 41(2):253-264.

Leung, S. C. H., Zhou, X., Zhang, D., and Zheng, J. (2011). Extended guided tabu search and a new packing algorithm for the two-dimensional loading vehicle routing problem. Computers & Operations Research, 38:205-215.

Malapert, A., Guéret, C., Jussien, N., Langevin, A., Rousseau, L.-M., et al. (2008). Two-dimensional pickup and delivery routing problem with loading constraints. CIRRELT-2008-37.

Moura, A. (2008). A Multi-Objective Genetic Algorithm for the Vehicle Routing with Time Windows and Loading Problem, In Intelligent Decision Support - Current Challenges and Approaches, pages 187-201. Gabler Edition Wissenschaft.

Moura, A. and Oliveira, J. (2009). An integrated approach to the vehicle routing and container loading problems. OR Spektrum, 31:775-800.

Zachariadis, E. E., Tarantilis, C. D., and Kiranoudis, C. T. (2009). A guided tabu search for the vehicle routing problem with two-dimensional loading constraints. European Journal of Operational Research, 195(3):729-743.